Functional Analysis HW 4

Deadline: 6 Mar 2017

1. Let X be a normed space.
   A sequence (x_n) in X is said to weakly converge to an element $x \in X$ if $f(x_n) \to f(x)$ for all $f \in X^*$. In this case, x is called a weak limit of the sequence (x_n).

   (i) Using the notation as above, show that if x and y both are the weak limits of (x_n), then $x = y$. Thus the weak limit of a sequence is unique.

   (ii) Show that if (x_n) is a sequence in a closed **subspace** M of X and x is the weak limit of (x_n), then $x \in M$.

2. Let $X = c_0$ and $M := \{e_n \in c_0 : n = 1, 2..\}$, where (e_n) is the natural base for c_0, that is

$$e_n(i) = \begin{cases} 1 & \text{if } i = n \\ 0 & \text{otherwise.} \end{cases}$$

   (i) Show that M is a closed **subset** of c_0.

   (ii) Show that the sequence (e_n) converges to 0 in c_0 in the sense of Question 1.
   (Thus the Question 1 (ii) does not hold for general closed **subsets**.)